

AP Calculus 2012-09-13

2.5/25a $f(x) = \begin{cases} 7x-2, & x \leq 1 \\ kx^2, & x > 1 \end{cases}$

find k to make $f(x)$ continuous at every point.

a) does $f(1)$ exist? **yes!**

b) $\lim_{x \rightarrow 1} f(x)$ - does this exist?

$$\lim_{x \rightarrow 1^-} f(x)$$

$$= \lim_{x \rightarrow 1^-} 7x - 2 = 5$$

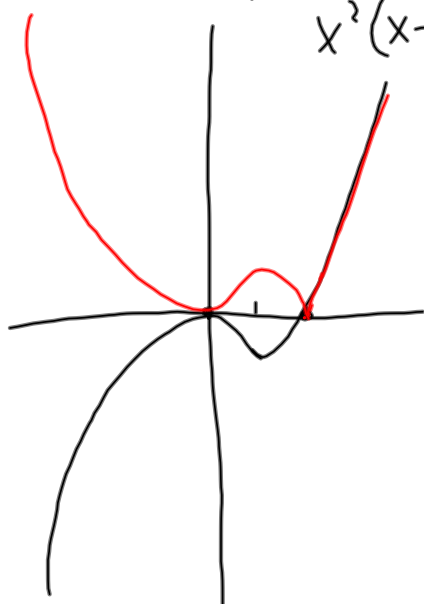
$$\lim_{x \rightarrow 1^+} f(x) = \lim_{x \rightarrow 1^+} kx^2$$

$$= k$$

the two one-sided exist,
but the two-sided limit exists
only if $5 = k$

c) does $f(1) = \lim_{x \rightarrow 1} f(x)$ [when $k=5$]?
yes.

2.5/21 a) $|x^3 - 2x^2|$ is this f^n ever discontinuous?
 $x^2(x-2)$



2.5/15 $f(x) = \frac{x}{x^2+1}$ is it discontinuous anywhere?

Types of discontinuities and SOME particular ways to get them?


1) Removable ^{ex} $\left(\frac{x-a}{x-a}\right) \cdot f(x)$
Disc...

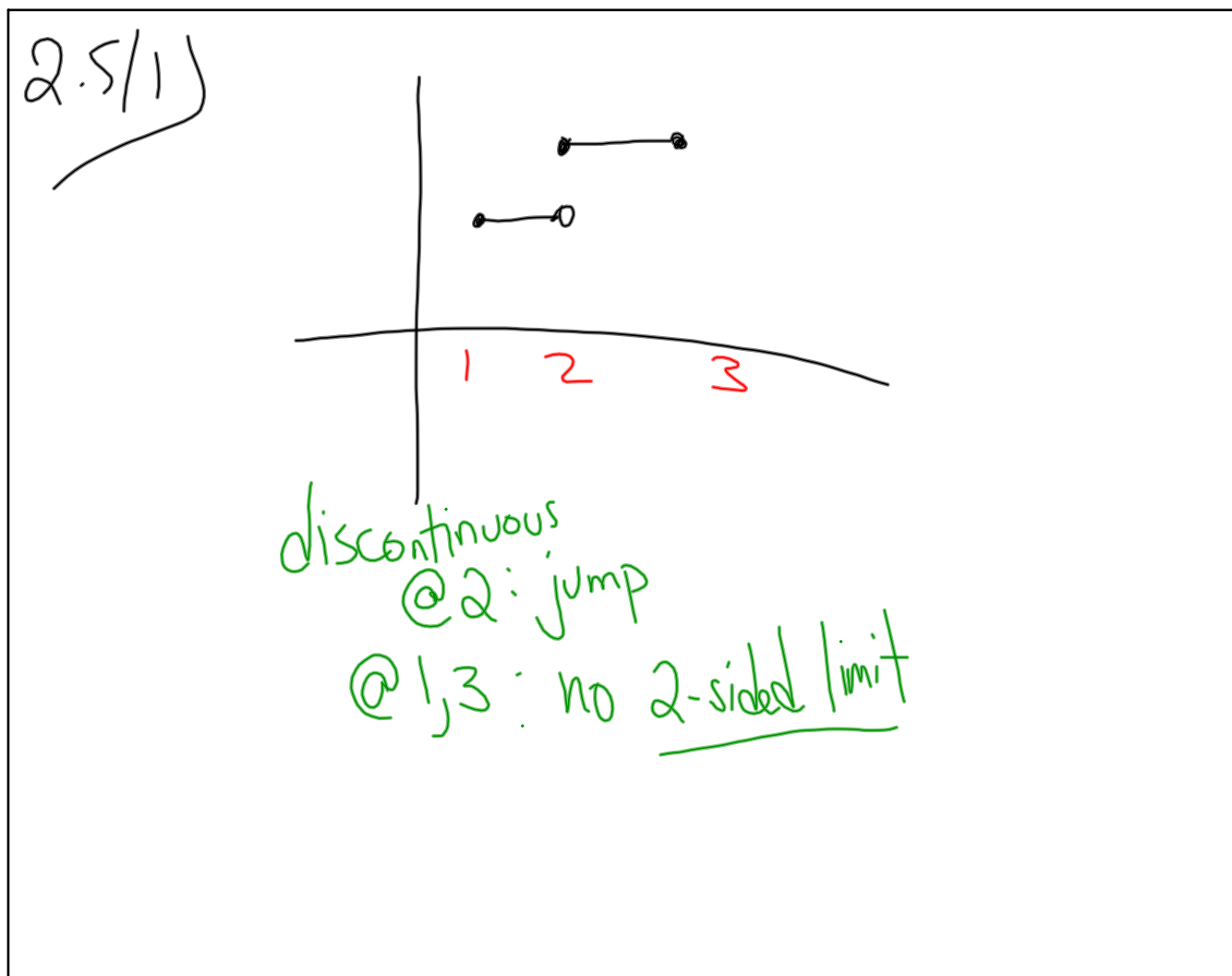
2) jump disc ^{ex} piecewise $f(x)$ [where pieces are joined together]

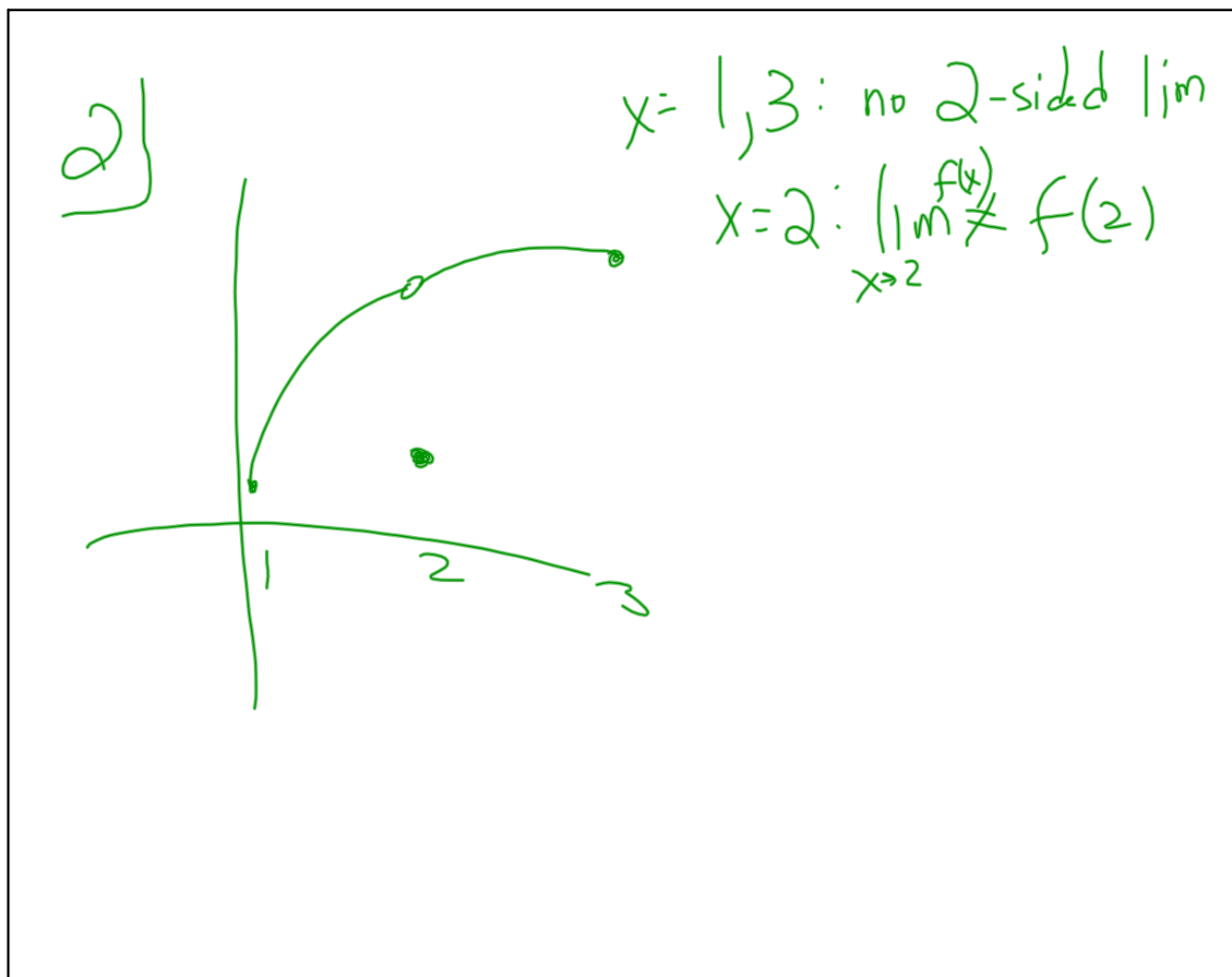
3) infinite [vertical asymptotes]

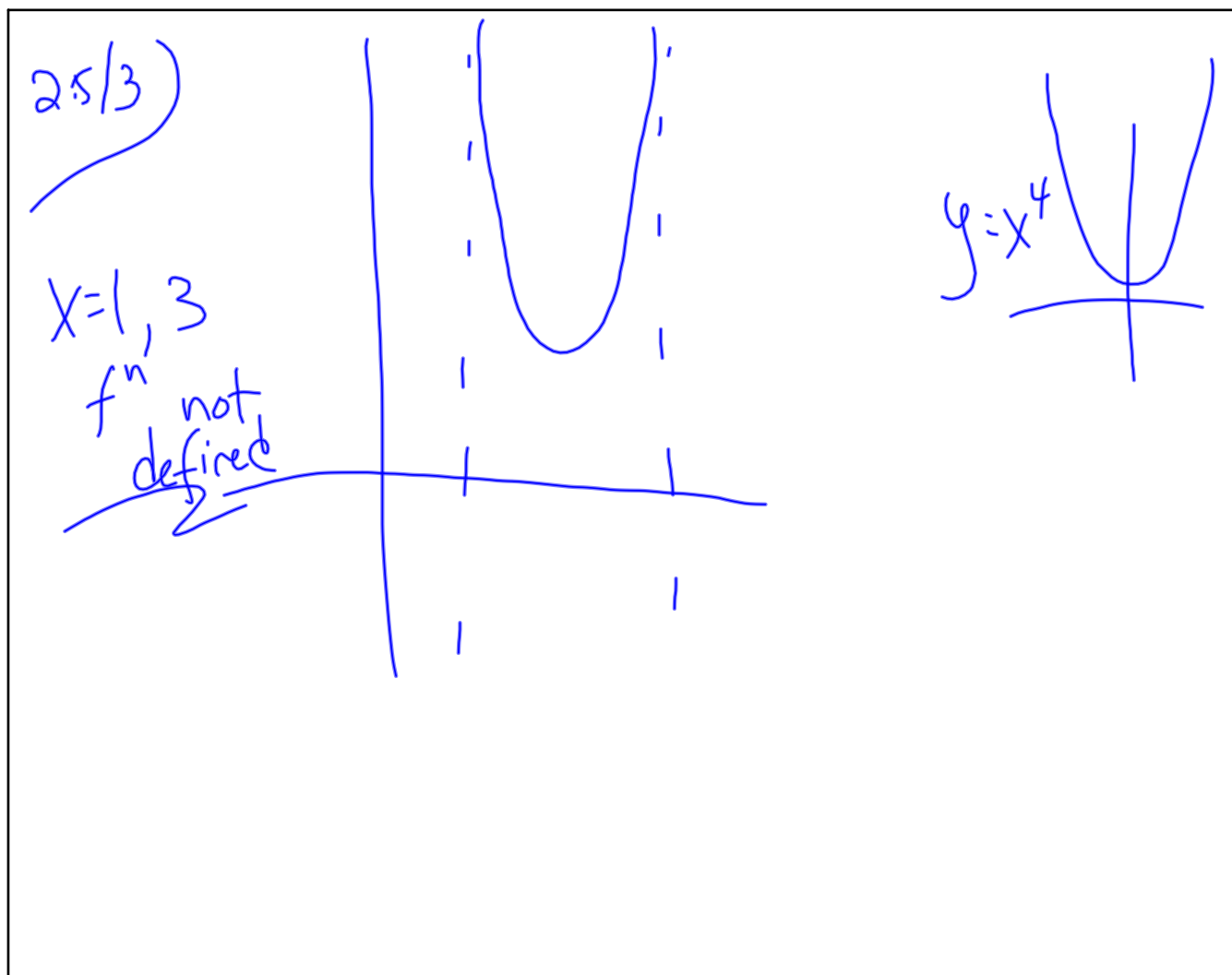
Zero in denominator
(that doesn't "cancel")

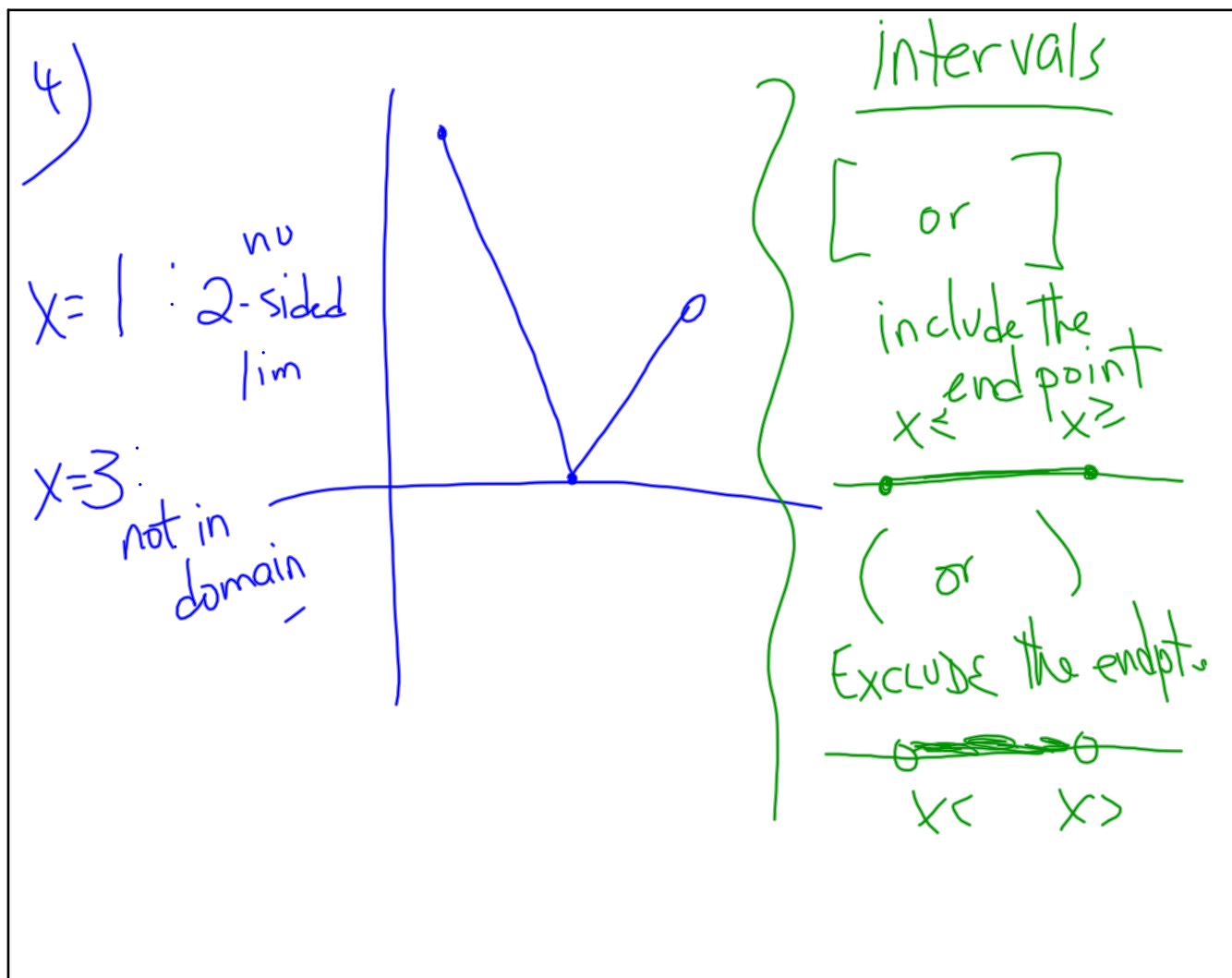
is x^2+1 ever 0?











25/25b)

$$f(x) = \begin{cases} kx^2, & x \leq 2 \\ 2x+k, & x > 2 \end{cases}$$

a) $f(2)$ exist? yesb) $\lim_{x \rightarrow 2} f(x)$

$$\lim_{x \rightarrow 2^-} f(x) = \lim_{x \rightarrow 2^-} kx^2$$

$$= 4k$$

$$\lim_{x \rightarrow 2^+} f(x) = \lim_{x \rightarrow 2^+} 2x+k$$

$$= 4+k$$

so...
2-sided limit exists ONLY when

$$4k = 4+k$$

i.e. $3k = 4$ or $k = \frac{4}{3}$

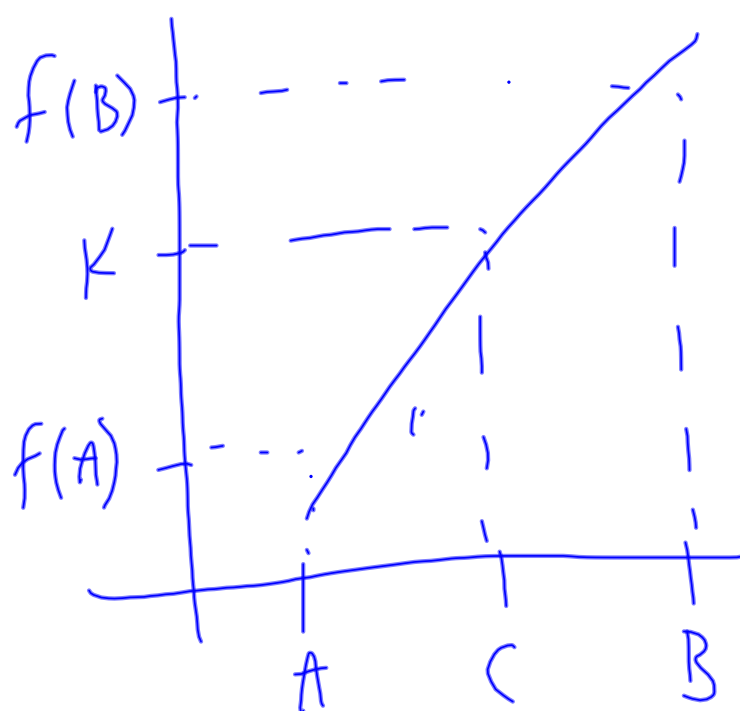
c)

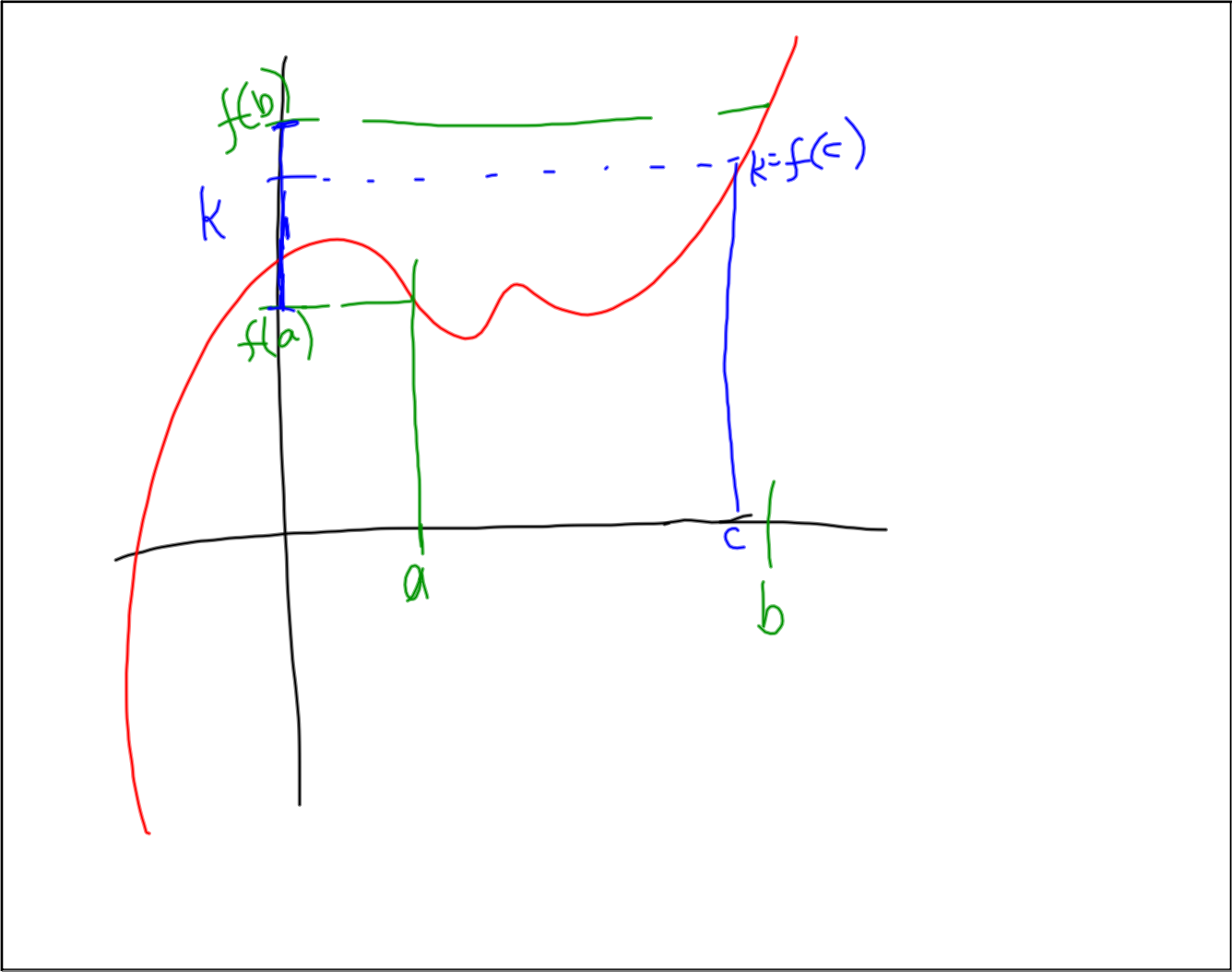
2.5] Intermediate Value Theorem (IVT)

Preamble $f(x)$ continuous for every $x \in [a, b]$

If $k \in [f(a), f(b)]$ (or $[f(b), f(a)]$ if appropriate)

Then $\exists c \in [a, b]$ where $f(c) = k$.





2.6/ limits of trig functions

→ limits of trig f^{ns} are obvious.

→ what is NOT obvious, must be remembered

$$\star \lim_{x \rightarrow 0} \frac{\sin x}{x} = 1$$

$$\star \lim_{x \rightarrow 0} \frac{1 - \cos x}{x} = 0$$